Towards a general many-sorted formal framework for describing certain kinds of legal statutes -PuDo-[1]

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Introduction

The 19th and 20th centuries in European continental and Anglo-American philosophical tradition showcase shifts in the paradigmatical descriptions of knowledge production, from given categories of perception of an empiric subject with the ability to synthetise and deductively order phenomena representing materia, with the potential to appear as objects[4]; to an empiric and cognitive subject with the ability to subsume and conceptualize the internal determinations of a subject matter, with the potential to appear in itself and to reveal itself as phenomena[5]; to approved natural scientific methods[6]; and via mathematisation of the validity of argumentation in natural languages[7] to different combinations of logical grammars[8] and theories of supporting matter[9]. Each construction carrying a specific notion of "referred" and "reference"; a manner of recognition, gaining access to, and organisation of, the referred; as well as a cohesive and authorative frame and a notion of nature, appearing due to the specific

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[4] Transcendental idealism - *categories* [34, B131, p. 153; 34, B138 pp. 156-157], *subject* [34, A350, p. 153], *phenomena* [34, A108, p. 137], *Verbindung* [34, B131, p. 152; 34, B134, p. 154], *Einheit* [34, B131, p. 152], *object* [34, A108-109, p. 137].
[5] German idealism - *subject* [25, p. 10]; *being, being-in-itself – being-there - being-for-other* [26, vol .1, book 1, section1, chapt. 2, B(a), p. 120; 26, vol .1, book 1, section1, chapt. 2, A(a), p. 110]; *the True - the absolute Idea - Idea - concept - science* [26, pp. 536, 842-843]; *law – phenomenal world* [26, p. 503].
[6] The construction of a universal natural scientific truth-criteria inspired by the refined developed mathematical and empirical tools applied for the validation, mapping and prediction of natural phenomena (naturalism) [15, p. 62]; and a theory of concepts based on the dynamically evolving expressive power of mathematics capable of projecting empirically verifiable relations and identities to an obtuse materia (philosophy of symbolic forms) [13, p. 45-85].
[7] Mathematical logic [22] and [45]. Logic positivism, analytical philosophy, linguistic phenomonology, structural linguistics, transformal generative grammatics [23, p. 14-15].
[8] The use of methods from different mathematical disciplines such as topology and transcendental algebra to construct logical theorems in theory of models.
[9] Epistemological models based on an intuitive empirism: naturalism [59, p. 83] and phenomenology - also referred to as representationalistic or correlationalistic constructions [52, p. 324]. Non-empirically founded epistemological models: modern philosophical idealism, minimal phenomenology, liberal naturalism [61, p. 402 f.], naturalistic monism - as a consequence non-representationalistic or non-correlationalistic constructions. The distinction between a contemporary Anglo-American tradition and a contintental European tradition: naturalism vs. naturalistic monism [81].

requirements for access and identification.[10] The analysis of the validity of arguments[11] including the references parallely went through an alteration, from the lifting up of entities and letting them present certain properties or constants and relate them with each other through logical connectors, validated empirically by the application of a so called truth-table[12]; to an expansion of the recognised properties and constants with a relational symbol with the expressive power from set-theory, validated through the distribution of entities in a certain given domain[13]; to the expansion of the formal syntax with a formal validation, by the application of a model in consideration, i.e., a formal structure which carries the 'meaning' of the purely syntactic propositions[14].[15]

As the legal activity includes the attributition of qualities and relations to entities in a legal context, as well as the issue of coherence and legitimacy of the legal systems, the reference-

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[10] See the notions developped for the dissulotion of a representational paradigm in favour of a non-representational one, conditioning different referential activities (mistake of direction as oppose to mistake of the senses [39, p. 52-53]). In a representational model: Reproduction (REP), Reference (REF), Preposition (PRE), Organisations (ORG), Nature (REP.ORG), Frame, Aggregation, Mini-transcendence, Meta-dispatcher. In an non-representational model: Attachment (ATT), Nature (ATT.ORG), Network (NET), Value (VAL), Scaling. [39].

[11] A formal system which consists of a set of synthactical expressions, constituted by well formed formulas, a finite sequence of symbols from a given alphabet which is part of the formal language: logical constants, the logical variables, and the auxiliary signs (terms). A formal language can be identified through a set of formulas in this language. A formula is a syntactical object which can be given semantic meaning through interpretation. [23, p. 25].

[12] In propositional logic there are logical connectors: „and" (conjunction, ∧), „or" (disjunction, ∨), „if…then" (implication, →), „if and only if" (implication (equivalence), ↔); as well as „not" (negation, ¬); logical variables – propositional letters, propositional variables; and auxiliary signs - brackets. Sentences, formulas – variables and composite expressions formed from the variables combined with the connectors. In addition, a function in this context is the attribution of a value to each entity of a specific kind, in this case unary functions mapping formulas onto truth values [23, p. 25]. Finally, the validity of the composite expressions depends on the truth-values of its constituents variables and the specific logical constants, i.e., true-tables [23, p. 45].

[13] In predicate logic there is a richer formal construct than in propositional logic, which consists of logical connectors: as in the former fn., and quantifiers: the universal quantifier ('for all') (∀) and the existential quantifier ('there exists') (∃), always combined with a variable, e.g., (∀x), (∃y); an infinite set of variables x, y, z , …; individual constants a, b, c, …; functional symbols of arity n (greater of equal to zero), which can take as argument any term (which is recursively construct in terms of the constants, variables and functions); predicate constants or predicate letters P, Q, R, …; unary standing for the properties of entities P(a), and n-ary $Q_i(a_1,…,a_n)$ standing for the relations between entitites, also taking consideration of the order between the logical constants [23, p. 66 f.]. The well-formed formulas are generated in a natural recursive way starting with the atomic ones, which are the syntactic descriptions that m terms fulfil a particular m-ary relation. The satisfiability and validity of w.f. formulas is defined in a coherent way based on the (implicitly assumed) existence of a model for the primitive notions of set and a membership relation between them, satisfying, for example, the axioms of Zermelo-Fraenkel ZF (sometimes with the Axiom of Choice, ZFC) [43]. In this framework one can express the most important formal setting of modern mathematics, for instance, the noitons of sets, conjunction, disjunction, bi-conditional, element, principle of extensionality, subset, property, union, intersection, ordered pairs, (formal) geometrical and arithmetical notions and properties, among many others [23, p. 83-87].

[14] Formal (mathematical) semantics [45].

[15] Regarding the application of logical models in legal practise: syllogistic logic [19, p. 14]; predicate-, propositional- and dialogical logic [17, pp. 29-31].

referred amalgates and the models for argumentative analysis offered a palette for a possible reconditioning of the legal activity, i.e. an altered relation between theory and practice.[16]

In the 18th and 19th century, traditional legal concepts, such as ownership, originally concipated as representations of the inert ideal relations between objects predetermined in a mathematical context[17], were re-interpreted as relations between objects percieved in terms of categories of space, time, substance (or object) and causality, and deductively systematised[18]; or as subsumed and conceptualized representations of internal determinations of a subject matter[19]. The application of categories of perception respectively operations of subsumation and conceptualisation, installed a rational subject, bound[20], respectively not bound[21], geographically and historically, as a reflection of the constructed legal systems.

The legal theoreticians of countries late to the table of the great civil law codification work of the 19th and early 20th century witnessed the extensive validation, mapping and prediction of natural phenomena due to refined methods and equipment [9, p. 84]; the conceptualisation of the link between the expanding synthetised natural empirical phenomena and the dynamically evolving expressive power of mathematics[22]; as well as the mathematisation of argumentative analysis, initially combined with a representational epistemological notion of supporting

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[16] The description of the construction, interpretation and systematisation of legal means within the application of an identified knowledge-theoretical construction and logical model – i.e. the specific rational.

[17] The discursive application of mathematics to predetermined objects of mathematics, that is figures and numbers; and the inventive and originary gesture of making the axioms of mathematics intelligeble. (technè, dianoia, metaxu, geometry, arithmetic [56, book VI, p. 511, c-d]; dialectics, épistémè, nous, principle, idea – ideatum [48, pp. 53-69]). And a schemata for analysis of formally valid premise-conclusions arguments, in which a conclusion is drawn from two premises, allowing a limited options of subject-predicate propositions (universal affirmative, universal negative, particular affirmative, particular negative). The terms acting as subjects and predicats refer to concepts and not to singular expressions and the validity of inferences are based on quantifying expressions, such as all and *some*, and not of the conjunction *if (...then)* – syllogistic logic [19, p. 14]. Regarding the application of the identified rational in a legal context: „Die Römer /…/ suchen die Einheitsforderung, der die Griechen in der Anschauung des Seins genügen wollten, im Reich des Handelns zur Geltung zu bringen. Und dadurch werden sie zu den ersten Logikern des Rechts." [9, p. 89]. See regarding critique of the ontological notion expressed in the the systematisation of legal rules by application of a syllogistic argument schemata [66, p. 813] – although there is no clear answer to the question whether the terms in a syllogistic logic are things (non-empty classes) or linguistic expressions for these things [19, p. 14].

[18] See regarding transcendental idealism fn. 3. Regarding the application of a transcendental idealism in a legal context:„Die reinen „Verstandesbegriffe", die „Kategorien", sind, nach Kants Definition, nichts anderes als die Mittel, mit deren Hilfe wir „Erscheinungen" /…/ buschstabieren, um sie als Erfahrung lesen zu können." „Die Rechtsbegriffen haben die gleiche Aufgabe der „Synthesis" zu vollziehen, aber ihre Einheitsbildungen haben einen ganz anderen Charakter, da sie sich nicht auf einen Einheit von Wahrnehmungen, sondern von Handlungen beziehen."; „Der Schwerpunkt des Problems verschiebt sich damit von der Wirklichkeit des Vertrags nach seiner „Möglichkeit." [9, p. 92-93 and p. 101].

[19] See regarding German idealism fn. 4.

[20] Legal constructions as the expression of the "Volksgeist" (Savigny, F. C. v., German historical school of jurisprudence) [53, p. 191, 202 and 206].

[21] Legal constructions as the expression of the "Geist" (Jhering, R. v., jurisprudence of concepts) [32] and [53, p. 313 and p. 315].

[22] Modern philosophical idealism [12] and [13].

matter.[23] In a number of works by different authors, considered to constitute a "period", the potential of different philosophical knowledge theoretical paradigms and models for argumentative analysis, as conditions for the legal referential activity, were investigated [41], [60], and [37 p. 299 and p. 302].[24]

2. Preliminary Considerations

2.1. Natural scientific veridiction criteria

The identification of phenomena by application of approved natural scientific methods engendered a philosophical conceptualized distinction between verified and non-verified referential content [63, p. 381 f.].[25] Carried over to a legal context the application of a natural scientific truth-criteria introduced the task of classification of the traditional legal concepts as scientifically respective non-scientifically viable.[26] The instauration of a physical existing world as an intersubjective validity implied the denial of the reliance on a rational subject or on an organisation of objects predetermined in a mathematical context, as a cohesive and legitimising level of foundation for legal constructions.[27]

2.2. Non-correlationistic concept theory

The philosophical project of conceptualisation of the formal expressive character of mathematics in natural sciences led to a replacement of the empirical and rational subject with a dynamically developping mathematical instance capable of projecting empirically verifiable relations and identities to an obtuse materia, whether in the context of natural sciences or with regard to the usage of symbolic forms in the context of social sciences.[28] The application of

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[23] Regarding correlationalistic, representationalistic, see fn. 7. Logic positivism [27] and analytical philosophy [70] and [72].

[24] The difference between the Scandinavian tradition and the North-American tradition [48, par. 28-30].

[25] The link to contemporary scientific naturalism [61, p. 399].

[26] Hägerström, A.'s „theory of correspondence" [42, p. 232 f.].

[27] Hägerström, A.'s „value theory and value psychology", often referred to as value nihilism [6] and [47, p. 105 f.].

[28] Cassirer, E. developed a theory of concepts based on the replacement of the transcendental categorical level of synthetisation with a mathematical instance, characterised by its dynamic development, its relative, non-representionalistic nature, supported by Einsteins theory of relativity, and the theoretical limit of its power of expressiveness. The character of mathematical empirical representation was further developped to a description of the function of symbols in natural language and as tools in the human sciences. The theory, placing natural sciences and social sciences on the same level, was labelled „Modern philosophical idealism" and „Philosophy of symbolic forms" [13, p. 45-85].

the theory of concepts in a legal context, implied the re-identification of legal concepts as non-correlationistic symbolic forms[29] corresponding to the different distinguished uses of natural languages – the *expressive function*, with an empirically verifiable meaning content deducible from the specific context, rendering a *substantial concept*; [30] and the *representative function*, with a meaning content deducible from the specific theoretical context, rendering a *functional concept*[31].[32] As the transcendental level of a rational subject was replaced with a dynamically evolving formal instance, the characterisation of the power of expressiveness of the formal mathematics as theoretically converging towards a limit, was further introduced as a foundational basis of the legal systems.[33]

2.3. Logic-mathematical model for argumentative analysis

A subsequent attempt involved a turn towards the movement which, in the wake of Freges "A logico-mathematical inquiry into the concept of number" [22] investigated the inner logic or deepstructure of natural languages through a comparison with the behaviour of mathematical number theory - initially paired with a representational epistemological notion, and an informal inference rule [22], [70] and [72].[34] Applied to legal theory, the instauration of the richer formal construct of predicate logic[35], implied a re-identification of traditional legal concepts, and construction, systematisation and interpretation of legal propositions through the translation into a mathematical terminology, of n-ary predicate symbols[36], constants and quantifiers.[37] The combination of an empiric verification criteria, a representational

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[29] Concepts in natural languages given meaning through the relative, non-correlationistic, signative function (Bedeutungsfunktion) [12, p. 48] and [9, p. 85].

[30] The expressive function of natural language (Ausdrucksfunktion) – substantial concept (Substanzbegriff) [12, p. 52].

[31] The representative function (Darstellungsfunktion) – functional concept (Funktionsbegriff) [12, p. 57].

[32] The legal concepts having a representative function, were thus not considered as fixed to an idealistic or materialistic reference-referred amalgate, but also not discarded as non-scientific [71, p. 303].

[33] Legal positivism. [24] The relation between the theoretical limit of expressivess of mathematics, employed in Cassirers theory of concepts, and Kelsens Grundnorm [5] and [63, p. 213 and p. 214 fn. 77]. Critique of the introduction of a contemporary idealistic notion as foundation for the legal system [65].

[34] See fn. 5. Verificationism as inference rule in a mathematical logical context, i.e. an informal semantics. An empirical epistimelogical notion in the context of mathematical logic: logical positivism [27]. Decline of logical positivism [23, p. 24].

[35] See fn. 13.

[36] The relational forms like: a R a (a bears the relation R to a; there are also ternary relations and so on) are acknowledged [22, §§ 70-72, 81-84]. Relations had previously in syllogistic and propossitional logic not been considered as fundamental as properties, for example set-theoretical properties: sets, conjunction, disjunction, bi-conditional, element, principle of extensionality, subset, property, union, intersection, ordered pairs [23, p. 83-87].

[37] The introduction of a formal syntax in the legal activity: cumulative plurality of legal consequences; disjunctive plurality of conditioning facts; semantic reference; use of indicative relational symbols to express

epistemological notion and a formal mathematical ordering instance, distributing qualities and connecting individuals in different contexts; instaured market, society and state as a frame providing coherence between the different scripts, and setting up a list of defined interests guiding the constructions and interpretations of the legal propositions.[38] Leaving to other disciplines, such as psychology, anthropology and sociology to study the motivation of the inviduals acting in and in relation to the legal scripts.[39]

2.4. The non-correlational epistemological functional concept vs. the logical relational symbol

The philosophical concept of symbols with an indicative-, expressional- and representional function, and the logical mathematical n-ary predicate symbol, share the endorsement of a formal instance regulating the distribution of relations between entities.[40] However, the relational logical symbol is a syntactic figure with the expressiveness of its set-theoretical provinence; in analytical philosophy connected with an intuitive empirical epistemological and semantical notion.[41] The function symbol on the other hand is a philosophical knowledge theoretical concept proposing a non-correlationalistic symbolic notion, conditioning all type of referential activities.[42][43]

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transfer of rights [66, pp. 820-823]. See further regarding the relation between logic and law [17, chapt. 3] and [54, chapt. 26 and chapt. 27].

[38] The denial of the reliance on a rational knowing subject or on an intuitively accessible known object as a cohesive and legitimising level of foundation for legal constructions, but the maintance of the existing world as an intersubjective validity, instaures quantifiable economical and inter-social factors as universaly valid rectified interests, part of the weighing of values underlying the legal constructions. – "Policy arguments in a rule based system" [2, p. 74 f.] and [44, p. 36] See the socio-anthropologic knowledge-theoretical terminology for the introduction of certain universally valid rectified interests, as opposed to the introduction of a fully transcendental foundational level: „mini-transcendence" and „metadispatcher" as opposed „maxi-transcendence" and „dispatcher" [39, p. 399-401].

[39] The epistemological veridiction criteria composed from the organised verified references from all type of scientific activities, provides the individual with a verified world, and makes it possible to construct a knowledge schedule to predict the values guiding a certain individual in the performing of a certain task, for example a judge. – Theory of „predictionism" for judges [55, p. 262 f.]. Social constructivism [2]. The adaptation of Ross' epistemological program in a Quinean replacement naturalism [28, pp. 8-10].

[40] See fn. 6 and fn. 7.

[41] See fn. 33. The Polish logician/mathematician Alfred Tarski, introduced the formal semantics in mathematical language, which was subsequently introduced for natural languages, and the consequent distinction between language seen as object of discussion, the object-language, and language as the medium in which such a discussion takes place (i.e., the meta-language) [74, section I.9]. For the application of formal semantics for natural languages see [24]. The separation between object- and meta-language also dissolved the problem posed in predicate logic, as developed by Gottlob Frege, and later in the analytical philosophy, by for example Betrand Russel and Peter F. Strawson, regarding the relation between the signs and their references, and the consequent distinction between denoting, referring and non-referring, non-denoting signs [70] and [72].

[42] See fn. 19 and fn. 20.

2.5. Continued development of the legal rational

As we have seen, each combination of epistemological paradigm and argumentative analytical model, when entangled and applied as conditions for the legal referential activity reveals its own rationality - the implication and potential therefore has to be assessed independently.[44]

Granted the opportunity to look back at last decades rash development of information products, software programming, and algoritms as its controlling DNA, it was neither the introduction of a natural scientific verification criteria, nor a non-correlationistic[45] concept theory, but the acknowledgement of the force in formalisation of natural language with the help of logic, - with an interest in the empiric dimension only as a semantical criteria of recognition[46], and indirect as an epistemological notion[47] -, that had the potential for changing the process of constructing legal networks, and the veridiction procedure for propositions within these.[48]

An introduction of a formal logic as the meta-language for the legal activity implies, besides the challenge in mastering the technical difficulties, also, a potential change in the theoretical understanding of the linking of the legal propositions with the non-legal content from different contexts, i.e. the creation of legal means - the identifying trait of the legal activity.

In the pairing of a full formal logic with a non-empiric epistemological paradigm, the notion of the creation of a reference through an ascending from a sensory particular[49], whether assisted by an object predetermined in a mathematical context[50], a knowing subject[51], or a mathematical instance[52], could be replaced with a procedure in which a speculative materia through a hiatus is attached to a reference, according to established methods in different type

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[43] The belonging of the theoretical framework suggested in the article [66] is uncertain, the terminology and the examples used suggest a focus on the mathematical logical aspect of the construction, interpretation and systematisation of legal means [66, p. 820-823] and [54, pp. 697-737], where as the critique of the ontological aspect of earlier in the legal theoretical environment applied representational epistemological notions, materialism [66, p. 813] and naturalism [66, p. 819], suggests a focus on the epistemological functional symbols of the non-correlationistic philosophy of symbolic forms (fn. 26 and fn. 27) [55, p. 262], [69, p. 74] and [71, p. 303].

[44] See fn. 4.

[45] See fn. 7 and fn. 28-32.

[46] Interest as empiric criteria of recognition in cognitive science and in neuro-psychology [21, p. 66 f.].

[47] Naturalism and phenomenology [61, p. 393].

[48] The development of Artificial Intelligence in law practice [26]. Large scale data analysis – data-centric legal systems [8]. The relation between logic and law [17, chapt. 3] and [54, chapt. 26 and chapt. 27]. The critique of the period [51].

[49] See fn. 9. Also referred to as *quasi-objects* [39, p. 426].

[50] See fn. 17.

[51] See fn. 4 and fn. 5.

[52] See fn. 5. See further, the compatibility of the empirical subject and a naturalism paradigm – naturalizing phenomenology, phenomenologizing nature [61, p. 394].

of referential activities[53], and linked to other references in accordance with the applied mathematical model[54], and with the application of an empirical criteria of recognition.[55] In this way the identifications are losened from any intrinsic properties and allowed to play different roles in different scripts, in an actor-network type of relationship.[56]

It would thus be possible to combine an logical n-ary predicate symbol with a non-correlational epistemological notion, in order to fix a one-to-one correspondence between typographical and semantic properties in a certain context [52, p. 332].[57]

Our aim is not to expand on this theoretical belonging, but to high-light one aspect of the discussed legal "period", the acknowledgement of the extended formal construct of predicate logic, in order to create a link to the full formal logical context.[58]

In this article, the first part will serve as a demonstration of how a certain choice of meta-language conditions the legal referential activity. We will introduce the creation, systematization, and interpretation of legal means, through the use of a formal syntax and a distributive validation criteria. In a second part we will add a formal semantics, in a third part we will see if the extended meta-language suits the legal activity, by the formalization of a specific legal statute regarding purchase of immovable property.[59] Lastly, in the final part, we discuss the potential advantages that this new formal deductive framework has for implementing initial artificial devices being able to enhance the deductive abilities of legislatives and researcher in laws.

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[53] Regarding the notion of network and value: „/…/ under the term „network" we must be careful not to confuse what circulates when everything is in place with the set up involving heterogenous elements that allow circulation to occur." [39, p. 32] The characterisation of the legal referential activity: „To be sure, all the connected elements belong to different worlds, but the mode of connection, for its part, is completely specific /…/." [39, p. 38].

[54] See fn. 8.

[55] The dissolution of the cohesive and authorative frame and the "essentialistic" notion of nature in correlational, representionalistic, epistemological paradigms through an analyze of "category mistakes" - mistake of direction as oppose to mistake of the senses [39, p. 52-53]. The instauration of an immanent non-representational reference-referred amalgate, and hence a non-correlational notion of nature [52, p. 324]. The possible compatibility between an empiric subject and a non-correlational paradigm [61, p. 407 f.].

[56] The theoretical conditioning of different referential activities in a non-corretionalistic, non-representational, epistemological paradigm - sociological relationalism [3], [14] and [40]; Actor–Network-Theory [37]. The conditions for the legal activity by the application of a formal logic paired with a non-empirical epistemological paradigm [39, chapt. 13 and 14].

[57] In formal logic this is achieved through the application of a formal structure, in the form of a chosen model, carrying the meaning of the purely syntactic propositions.

[58] A logic combining a formal syntax and formal semantics.

[59] Following elements of the logic-based approach to creation of a logical formalization of a given legal text has been identified: „(1) legal texts have a determinate logical structure; (2) there is a logical formalism sufficiently expressive to support the full complexity of legal reasoning on these texts, and (3) the logical expression can achieve the same authoritative status as the text from which it is derived." [8, p. 8].

2.6. Introduction of constants, variables, functions, relations and quantifiers

The advantage with the extented logic vocabulary of predicate logic, as developed by Gottlob Frege, and later by representatives of the analytical philosophy, such as Bertrand Russel and Peter F. Strawson, is that it offered terminology for dealing with how objects are used as semantic support[60] for a textual formal context[61], and how certain kinds of such objects can be chosen to represent specific "qualities" of objects[62], and can be related to each other[63] in order to create textual/factual distinctions within a unified context.[64]

Carried on to a legal field, the *objects/subjects (i.e. entities)* which support the unified factual/textual-physical context of a legal system are persons, objects and relations between these,[65] taking place in specific spatial/temporal events.[66] The function of the legal system is to support certain type of interactions by lifting up certain *objects/subjects (i.e. entities)*, let them represent certain properties and relations (in a logical sense) gather them under a "sign" (frequently referred to as a legal consequence, or as a logical thesis within a formal implication) and relate them with each other, through logical connectors like implication, conjunction or equivalence, in the form of '[conditions] implies [consequences]'.[67] The 'systematisation' of a legal system consists in grouping the network of "logical sentences" related through logical connectors in the form of: Hypothese implies Theses.[68]

The collection of specific *objects/subjects (i.e. entities)* that are "lifted up" to support a property or a relation[69], and to constitute a part of the factual/textual context of the legal

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[60] See fn. 37.

[61] This notion can be understood as the model in consideration, i.e, the formal structure which carries the 'meaning' of the purely syntactic propositions. It is worth to notice here that the models themselves are formal structures and usually they are base on the existence of a kind of 'canonical' model, such as the existence of sets and a membership relation obeying the axioms of Zermelo-Fraenkel Set Theory with Choice [45].

[62] Properties in a logical sense.

[63] Relations in a logical sense. [22, §§ 70-73, 81-85].

[64] The "model", constructed from the propositions in the object language, in our case the legal system.

[65] See Fn. 5.

[66] In predicate logic and later in the analytical philosophy, the interpretation of signs after the translation into the logical language, depends on the distinction between the character of the signs as denoting-/referring resp. non-denoting/non-referring. [70, p. 479] and [72, p. 321-324] In contrast, in classical first-order logic the 'models' turn out to again abstract entities like the natural, rational or real number, for example.

[67] See fn. 13.

[68] See fn. 13.

[69] In a logical sense.

system, depends on how certain identified interactions, of which the property or relations are part, should be promoted.[70]

In accordance with the formal logical vocabulary introduced above, a legal *expression*, such as a "right ", can be employed either as a *property*[71], regarding a specific feature that a collection of objects possesses', in this case the type of use of an object, or as a reference to a specific relation[72] between several parties involving the achievement of such a property.[73] In the first case, the right is defined as the competences regarding the use of a specified object [66, p. 821], supported expressly or implicitly by the specific legal system. In the second case, the specific relation [66, p. 820] and the underlying logical theses have to be identified, before the validity of a statement containing the expression can be tried [66, p. 823].

We have so far introduced a part of the extended syntax, but on an semantic level, the empirical distributive validation procedure remains. What we want is the expansion of the argumentative analytic model with a formal validation criteria, applying a formal structure, in the form of a chosen model, carrying the meaning of the purely syntactic propositions.[74]

2.6. Many-sorted first-order logic: Introduction of an extended logical formalism

In classic many-sorted first-order logic (MFOL) [43, chapt. VI] we consider several entities like sorts, constants, variables, functions and relations structured by means of sound logical connectives and (universal and existencial) quantifiers. Now, the models in this framework have again a very abstract and linguistic nature since they emerge within the context of the standard framework of Zermelo-Fraenkel Set Theory with Choice (ZFC) [45].

On the other hand, in our present case of finding syntactic-semantical formalizations of (purchase) laws, we need to be able to speak about suitable frameworks having 'models' which are located in the (physical) world, e.g. persons, properties, documents of purchase, among others.

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[70] The legal script delegates different roles to identified actors, in the form of '[conditions] implies [consequences]', according to the legislator's considerations [39, chapt. 13 and 14].
[71] In a logical sense.
[72] In a logical sense.
[73] The term could here refer to the cumulative "consequences" connected to the "condition" entering of a formally valid transfer agreement, that is, the reciproque claims of delivery and payment, the competence to transfer, and the competence to demand the object from a third party holding the object - the so called netto effects. [29, p. 725] The term could also refer to the, cumultative, or individual consequence and connected conditions, related to the protection towards certain identified parties, not being part of the transfer agreement.
[74] See fn. 61.

So, in our case, we pursue to develop an extended logic framework for enhancing the semantical scope of these formal structures to more pragmatic objects/subjects like those considered in laws.

3. Formalisation of the ransfer of Ownership of (Immovable) Property – PuDo

Our next task is to employ the introduced logical framework, to create a „model" for a chosen part of the unified factual/textual-physical context of the legal system. We have chosen the Swedish legal system, and specifically the statute for purchase of immovable property as our object of formalisation.[75] One of the reasons is that the legal statute is constructed in accordance with a logical framework that acknowledge the relational form.[76] In addition, each of the notions involved in purchase laws like (immovable) property, purchase, buyer, seller, purchase document and the obligations between the parties; together with their formal interrelations seems to fit quite well into the deductive behaviour of a many-sorted first-order logic framework. Furthermore, it allows us to enhance the semantic scope of the formal structures by specifying the vocabulary and the attributed content of each employed variable in the vocabulary, within the model.

3.1. Towards a new logic formal framework for describing (purchase) laws

In this section, we formalize in a new logic framework the following sentence "A purchase of real property is concluded through the drawing up of a document of purchase signed by the seller and buyer."[77] Let us denote by SENT the former sentence.

We will take inspiration of the well-known framework of many-sorted first-order logic [45]. A lot of aspects of this particular logic approach are suitable for our purposes because many-sorted first-order logic is based on a `multi-semantic' approximation, namely, the models for

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[75] The regulation of transfer of immovable property, is provided in a unified regime, where the the identified relations constituting subdivisions, are indicated in the heading of the different sections of the statute. "Purchase","Rights and obligations of the seller and the buyer", "Priority on grounds of title registration", "Bona fide acquisition by virtue of title registration and the import of title registration in certain cases." (Jordabalk (1970:974), Land Code, https://www.kth.se/polopoly_fs/1.476821!/Land_Code.pdf).
[76] See Fn. 16 and Fn. 30.
[77] Chapt. 4, Section 1. Land Code. We here take the Scandinavian perception of transfer of ownership as a starting point, see fn. 73.

the theory are, by definition, divided into a collection of `sorts'[78] which can model more easily the diversity of entities that appears naturally in the syntactic description of (purchase) laws, e.g., real and movable properties, sellers, buyers and contracts. Besides, the temporal aspects will be very important in our formalization, therefore we will use also a temporal sort. In addition, all the sorts defined here will have an implicit temporal component in the sense that they are considered in present time. For instance, if we say let A denotes the sort representing the collection of all buildings in Stockholm we mean by that the collection of all buildings in Stockholm in the present.

Now, since the interpretations in our particular case will be given by concrete entities existing in nature (like people and properties), we will call the logic emerging here *physical multi-sorted first-order logic (PML).*

So, let us start fixing some initial terminology: Let us denote by P_{nl} the collection of all natural or legal persons with certain well-defined legal capacity[79]; by Pr_R the collection of all real properties (in Sweden), i.e. well-defined pieces of land together with the fixed constructions inside of them[80]; by Pr_M we mean the collection of movable properties, i.e., here all entities in Sweden are virtually included except persons and real properties[81]; by Pr we denote the union of Pr_R and Pr_M, i.e. the collection of properties in Sweden; by $w: P_{ph}(PrR)$, $w: P_{ph}(Pr_M)$ and $w: P_{ph}(Pr)$, we mean that w is a finite collection of real, movable and (both types of) properties, respectively. Here we use the functional symbol $P_{ph}(-)$ to indicate the similarity of this sort with the set-theoretic construction of the power set of a fixed set, i.e., the set whose elements are the subsets of the corresponding set.

Besides, we use the sub-index `ph' in order to mention the fact that we are dealing with a kind of `physical' and, at the same time, formal construction very closed related with the idea of considering sub-collections of entities. Moreover, we need an additional relation $\in_{ph}: Pr \times P_{ph}(Pr)$, used as follows: $s \in_{ph} w$ means that *the property "s" belongs to the collection of properties "w".*

In addition, we use a sort T for describing the temporal dimension of the events, for instance, an interpretation of this sort could be the classic mathematical notion of positive real numbers

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[78] For the present case, sorts can be understood as the conceptual unities which constitute the formal taxonomy where the different 'legal' actions take place.
[79] Chapt. 11, Section 1. Code of Judicial Prodedure (1942:740).
[80] Chapt. 2, Sections 3-5. Land Code.
[81] Property, movable property, immovable property [30, p. 30 and pp. 40-42].

$\mathbb{R}^+$. We will use also a binary relation for comparing the moments where events happen, i.e., $\leq_{ph}: T \times T$. Finally, we will denote by D the collection of contracts describing the purchase of one or more properties in Sweden.

Moreover, we need to define now the collection of relational symbols that we will employ in our description. First of all, we will denote the relation of purchase by $Pur: P_{nl} \times Pr \times P_{nl} \times T$. So, the expression $(a, s, b, t) \in Pur$ would mean that *at the time "t" the person "b" has purchased the property "s" from the person "a"*.

Now, let us define the 'purchase document' relation as follows: $PuDo: D \times P_{nl} \times P_{nl} \times P_{ph}(\mathrm{Pr}) \times T$, and $(d, a, b, z, t) \in PuDo$ if and only if *"d" is the document of purchase of the properties in "z" from the person "a" to the person "b", signed by both of them at time "t"*.

So, we can formalize our initial paragraph SENT with the notions already defined as follows:

$$(\forall a, b: P_{nl}) \left(\forall w: P_{ph}(Pr) \right) (\forall t: T)[(\exists v: Pr_R)\left(v \in_{ph} w\right)$$

$$\rightarrow \Big[(\forall s: \mathrm{Pr})(s \in w \rightarrow (a, s, b, t): \mathrm{Pur})$$

$$\leftrightarrow (\exists d: D)(\exists r: T)(\forall s: \mathrm{Pr})\left(s \in_{ph} w \rightarrow \left(r \leq_{ph} t\right) \wedge \left((d, a, b, s, r): PuDo\right)\right]\Big]$$

Let us explain in more detail the precise meaning of the former sentence.

First, by pragmatic reasons we assume that SENT speaks (implicitly) about all the possible persons who could be potential buyers and sellers in Sweden, which are essentially the collection of people in P_{nl}.

Second, SENT applies only for collections of properties which includes at least one real property[82]. Thus, we included a sub-paragraph mentioning this fact explicitly, i.e., $\cdots (\forall t: T)[(\exists v: Pr_R)\left(v \in_{ph} w\right) \rightarrow \cdots$.

Third, we assume that exactly at the time when the purchase document is signed by both parts, the purchase relation starts to be fulfilled between them, independently if in a future time the same properties will be purchased again.

Fourth, the central part of the sentence is just expressing the fact that a have purchased all the properties in w (which includes at least one real property) to b, if and only if there is one

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[82] "A purchase of real property is concluded through the drawing up of a document of purchase signed by the seller and buyer. The deed shall contain a statement of the purchase price and a declaration by the seller that the property is transferred to the buyer." Chapt. 4, Section 1. Land Code.

document of purchase which was signed by both parts at some time before (resp. immediately) when the purchase's relation is stated to be valid.

4. Towards an Arfiticial Co-creative Legal Assistant

One of the advantages of using a kind of many-sorted first-order logic grounding framework for characterizing legal statutes, i.e., propositions in a legal context, is that we can use very robust software like the Heterogeneous Tool Set (HETS) [49] in order to specify many-sorted first-order concepts, like the former ones, by means of the Common Algebraic Specification Language (CASL) [4]. Now, HETS has a lots of advantages for doing conceptual operations at a symbolic and semantic level. For instance, one is able to compute explicitly a formal conceptual blending of two (input) concepts by means of categorical colimits, where the commonalities can be codified by hand throughout a generic space [15]. Furthermore, HETS has also integrated consistency checkers for finding initial verifications/refutations of the (in-)consistency of the blended theories. This kind of tool can be very useful if one desires to compare logically similar (and very complex) laws from different countries (for example within the European Union), because one could codify them in a first-order logic language, compute the commonalities in terms of a generic space, do the corresponding formal blend in HETS and finally analize very carefully the final blended space, not only from a human legal perspective (perfomed by the researchers in laws) but also with the powerful syntactic deductive tools of HETS [78] and [79]. In particular, this would enhance considerably the logic scope and accuracy of the legislators and researchers in laws during their intellectual work.

5. Main Conclusions

The main results of this work aim to open a quite concrete new way of using the deductive power of classic formal logic frameworks (e.g. many-sorted first-order logic), together with the corresponding artificial realizations of (some aspects of) them (e.g. HETS), for improving, enhancing, and facilitating the research in and the creation of legal constructs at a global scale. In particular, one can do a similar formalizing procedure as before not only for swedish purchase laws, but potentially for any kind of legal statutes, by doing a suitable initial

description of the sorts-formalization. It is important to note that the legislator and lawyer are essential part within our proposal, since we are looking for improving their deductive capabilities instead of replacing them.

So, in conclusion, from this perspective formal logic and laws can work together in a new way allowing us to be able to construct artificial legal co-deductive assistants, which can improve, for example, the (purely theoretical) deductive skills of the legal practicioners and theoreticians of law.

Acknowledgements

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